1.4 - Inverse Functions

Skills

- Determine if a function is one-to-one
- Find the inverse of a function

One-to-One

A relation is a function if there is only one y value for every x value in the domain.

A function is one-to-one if there is only one *x* value for each *y* value in the range

To be a function, the graph must pass the vertical line test

Thus, to be one-to-one...

Checking Algebraically

Swap x and y

Solve for y

If you don't get a ± or absolute value, then the function is almost certainly one-to-one

Finding an Inverse Function

...is actually what I just described!
Once you've solved for y, you've got the inverse function.

Example

$$y = x^3 - 8$$

Example

$$y = \frac{2x+3}{x+4}$$

Properties and Notation

$$f^{-1}(x)$$

$$\left(f \circ f^{-1}\right)(x) = x$$

Their graphs are reflections across the line $y = x$

Example

$$f(x) = 2x + 6$$

$$g(x) = \frac{1}{2}x - 3$$

Are these functions inverses of each other?

Domain and Range

The domain of the function is the range of the inverse

Sometimes we must restrict the domain of a function in order to find an inverse

Example

$$f(x) = (x-2)^2$$

Can we restrict the domain of this function to make it one-to-one?