

Probability Review: Combining Random Variables

Skills:

- Find the mean and standard deviation of a linear transformation of a random variable
- Find the mean and standard deviation of a combination of multiple random variables

Linear Transformations

If I add seven to every value in the distribution, what happens to the center (mean)?

If I add seven to every value in the distribution, what happens to the spread?

$$\mu_{X+a} = \mu_X + a$$

$$\sigma_{X+a} = \sigma_X$$

Linear Transformations

If I multiply every value in the distribution by seven, what happens to the center (mean)?

If I multiply every value in the distribution by seven, what happens to the spread?

$$\mu_{bX} = b \cdot \mu_X \quad \sigma_{bX} = |b| \cdot \sigma_X$$

Linear Combinations

If I add two distributions, what would you hope would happen to their means?

$$\mu_{X+Y} = \mu_X + \mu_Y$$

Linear Combinations

If I add two distributions, what would you hope would happen to their spreads?

Alas, standard deviations don't add.

However, variances do add!

...but only if the variables are independent.

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$$

Example

A firm is planning an investment. The potential profit from this investment (in millions of dollars) varies according to the following distribution.

x	1	1.5	2	4	10
p	0.10	0.20	0.40	0.20	0.10

Example (continued)

The firm must immediately surrender 10% of the profits, plus an additional \$200,000 to its investment partners.

What are the mean and standard deviation of the profits after payments to the investment partners?

Example

The number of cars on a small ferry varies according to the following distribution.

Find the mean and standard deviation of total cars carried for fourteen trips by this ferry.

x	0	1	2	3	4	5
p	0.02	0.05	0.08	0.16	0.27	0.42