Some Special Random Variables

The Bernoulli Experiment

A **Bernoulli Experiment** is an experiment (activity) where:
[1] there are only two outcomes—success, and failure;
[2] each trial of the experiment is independent of any other trial; and
[3] the probability of success is constant.

Binomial Random Variables

The Definition

A **Binomial Random Variable** counts the number of successes in a fixed number (n) of trials of a Bernoulli experiment.

Mean and Standard Deviation

If B is a binomial random variable, then the following formulas give the mean and standard deviation of B.

Equation 1 - Mean of a Binomial Random Variable

$$\mu_B = n \cdot p$$

Equation 2 - Standard Deviation of a Binomial Random Variable

$$\sigma_B = \sqrt{n \cdot p \cdot (1 - p)}$$

Examples

[1.] A treatment for Pinworm infestation is 90% effective; the treatment will cure the infestation in 90% of cases. In a group of 20 randomly selected cases of Pinworm infestation, what is the probability that this treatment will cure exactly 15 cases? At least 19 cases?

First of all, notice that we have a fixed number of trials of a Bernoulli experiment—each case is one trial; each trial results in success (cured) or failure (not cured); the probability of success is constant (90%); each trial is independent (we assume; but this is a reasonable assumption). So the random variable X can count the number of cured cases (out of 20). The first question, then, is P(X = 15), which can be computed easily: $\binom{20}{15}(0.9)^{15}(0.1)^{5} \approx 0.0319$. The second question is P($X \geq 19$), which must be broken up into P(X = 19) + P(X = 20) in order to compute the answer (0.3917).

Geometric Random Variables

The Definition

A **Geometric Random Variable** counts the trial number of the first success in repeated trials of a Bernoulli experiment. In other words, the experiment keeps repeating until success is achieved, and the number of trials that it took is recorded.

Mean and Standard Deviation

If G is a geometric random variable, then the mean and standard deviation of G are given by the following formulas.

Equation 3 - Mean of a Geometric Random Variable

$$\mu_G = \frac{1}{p}$$

Equation 4 - Standard Deviation of a Geometric Random Variable

$$\sigma_G = \frac{\sqrt{1-p}}{p}$$

Examples

[2.] The (approximate) probability of matching some of the winning numbers in the Arizona Lotto is given in the following table.

Table 1 - Lotto Probabilities

x	**0**	**1**	**2**	**3**	**4**	**5**
P(X = x)	0.3713	0.4312	0.1684	0.0272	0.0018	0.0001

A player wins a prize only if they match at least 3 winning numbers. A very foolish player (probably from Nevada) decides to play one ticket each week until he wins. What is the probability that he has to play for a full year (52 weeks) to win? How many weeks should he expect to play in order to win once?

Notice that we once again have a Bernoulli experiment, this time with probability of success $p = 0.0291$. Let W represent the week number of the first win. The first question is $P(W = 52) = (0.9709)^{51}(0.0291) = 0.0065$. There is a 0.65% chance of winning after one year of play.

As for the second question—whenever we are asked about an expectation, that's a clue to find the expected value. Since $p = 0.0291$, the expected value of W is $\mu_W = \frac{1}{p} = \frac{1}{0.0291} \approx 34.36$.

This does not contradict the earlier result; the first question was the probability that you win *after* 52 weeks—not 52 weeks or earlier.